2025

MATHEMATICS — MINOR

Paper : MN-3

(Ordinary Differential Equations and Group Theory)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

All the notations and symbols have their usual meanings.

Group - A

(Ordinary Differential Equations)

Answer ***any nine*** questions.

1. (a) Solve : $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$.

(b) Show that the general solution of the equation $\frac{dy}{dx} + Py = Q$ can be written as $y = K(u - v) + v$, where K is a constant and u, v are its two particular solutions. 2+3

2. Define Integrating factor of a differential equation and solve : $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$. 1+4

3. Solve : $x\frac{dy}{dx} + y = y^2 \log x$. 5

4. Reduce the differential equation $x^2p^2 + py(2x + y) + y^2 = 0$ to Clairaut's form by the substitution $y = u$, $xy = v$ and solve it to find general solution and singular solution. 1+2+2

5. Solve $(D^2 + 1)y = 3\cos^2 x$, where $D \equiv \frac{d}{dx}$ by the method of D-operator. 5

6. Solve by the method of variation of parameters : $\frac{d^2y}{dx^2} + 9y = \sec 3x$. 5

7. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = (x-2)e^x$. 5

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8. Solve and find the singular solution of $xp^2 - 2py + 4x = 0 \left(p = \frac{dy}{dx} \right)$. 5

9. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$, given that $\left(x + \frac{1}{x} \right)$ is one integral (or solution). 5

10. Solve the following Simultaneous Linear differential equations : 5

$(D - 1)x + Dy = 2t + 1$

$(2D + 1)x + 2Dy = t$, where $D \equiv \frac{d}{dt}$.

11. Show that the family of confocal conics $\frac{x^2}{\left(a^2 + \lambda\right)} + \frac{y^2}{\left(b^2 + \lambda\right)} = 1$ is self-orthogonal, λ is a parameter. 5

12. Solve : $\frac{d^2y}{dx^2} + 2\sin x \frac{dy}{dx} + 2y\cos x = 0$. 5

13. Solve the equation : $(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$. 5

14. Solve the simultaneous equation : $\frac{x\,dx}{z^2 - 2yz - y^2} = \frac{dy}{y + z} = \frac{dz}{y - z}$. 5

15. Solve the differential equation : $\left(D^3 - 5D^2 + 7D - 3 \right) y = e^{2x} \cosh x \left[D \equiv \frac{d}{dx} \right]$. 5

16. Find an integrating factor of the differential equation $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$ and hence solve it. 2+3

Group - B

(Group Theory)

Answer ***any six*** questions.

17. Prove that the set Q of all rational numbers other than 1 under the binary operation '*' defined by $a * b = a + b - ab$ forms an Abelian group for all $a, b \in Q - \{1\}$. 5

18. Define order of a group. Also, explain the order of an element in a group and hence prove that each element of a finite group is of finite order. 1+1+3

19. Let G be a group and H, K are two subgroups of G. Prove that $H \cap K$ is also a subgroup of G. Is $H \cup K$ also a subgroup? Justify your answer. 3+1+1

20. Let $G = \left\{ \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix} : a, b \in \mathbb{R}, a \neq 0 \right\}$. Show that G becomes a group under usual matrix multiplication. 5

21. If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$ be an element of S_7, then find α^{-1}. Is it an even permutation? 3+2

22. Define subgroup of a group G. Let G be a multiplicative group and H be its subset defined by : $H = \{x : x \in G \ \& \ xa = ax, \forall a \in G\}$.

Prove that H is a subgroup of G. 1+4

23. Prove that every group of prime order is cyclic. 5

24. Show that the 8th roots of unity forms a cyclic group. 5

25. Prove that every cyclic group is a commutative group. Is the converse true? Justify your answer. 3+1+1

26. Prove that for a cyclic group G, there exists an element a in G such that $o(a) = o(G)$. 5
